AXIAL TEMPERATURE DISTRIBUTION OF A THERMIONIC CATHODE

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A practical method for calculating the temperature distribution along a thermionic cathode based on numerical evaluation of the integral of the heat-balance equation is described. The radiation component of the heat flux, the Joule heat, the heat conduction along the cathode, and the temperature dependence of the electrical conductivity are taken into account.

Plasmotrons with heated thermionic cathodes are very widely used in practice. The absence of an electrode spot and the diffuse nature of the attachment of the arc to the cathode have a decided effect on the operation of thermionic cathodes. The diffuse nature of the discharge from the cathode shows up best in an argon medium and somewhat worse in a hydrogen medium. The pressure also has a considerable effect on the diffuseness of the discharge. The current densities obtainable from thermionic cathodes can be as high as 10^4 A/cm^2, and the electric field strengths can be 10^5 V/cm (for hydrogen) [1].

In practical plasmotrons with sectioned channels, and also when using such plasmotrons to preionize the volume of a MHD generator (plasma-technological equipment [2]) the use of thermionic cathodes is extremely promising.

It is therefore of interest to develop a simple practical method of calculating the temperature gradients at points where the cathode is fixed, and in the arc attachment region, and also to calculate the temperature distribution along the cathode as a function of the plasmotron operating parameters.

Various approaches to the solution of this problem are described in the literature [3,4]. In the majority of cases, however, the solutions given are obtained by making considerable simplifications in the formulation of the problem, or have an extremely unwieldy form which makes them difficult to use in practical calculations.

In this paper we consider a somewhat different formulation of the calculation model and the boundary conditions.

Suppose the cathode is a tungsten rod fixed in a casing cooled in water. We will assume that an electric arc is "attached" to the top of a cathode, which fills the whole transverse cross section of the rod, which is attained under rarified-atmosphere conditions or a vacuum [1]. These conditions correspond to the case when the cathode is surrounded by a longitudinal flow of gas, and convective heat transfer can be neglected compared with radiation loss. This assumption holds for pressures in the plasmotron channel of up to 20 torr [1].

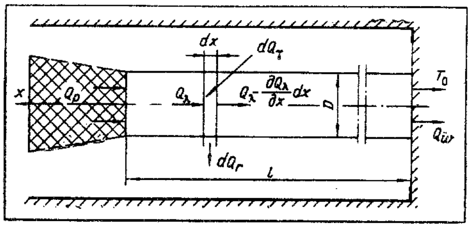

Fig. 1. Diagram of the cathode heat transfer.

Consider the heat balance of an element of the cathode consisting of a cylinder of radius r and length dx (Fig. 1). We will assume that the temperature over the radius of the elementary cylinder is a constant T(r) = const. As in [1], this assumption is justified in view of the large value of the

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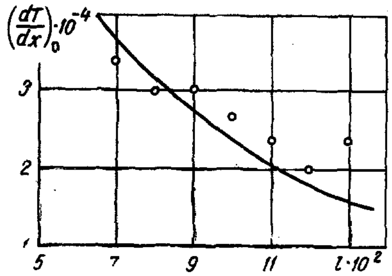

Fig. 2

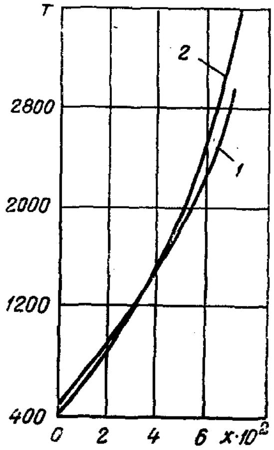

Fig. 3

Fig. 2. Comparison of the theoretical and experimental data for a tungsten cathode of diameter 0.012 m: the points represent experimental data, and the continuous curve is theoretical. $(dT/dx)_0$, deg/m; l, meters.

Fig. 3. Comparison of the theoretical curves of the temperature distribution (°K) along the length of a tungsten cathode of diameter 0.008 m: 1) temperature-distribution curve from [1], taking into account convective heat loss from the surface of the cathode in the case of a flow of hydrogen at the rate of 0.26 g/sec at a pressure of 20 mm Hg; 2) theoretical, using the method described here but ignoring convective heat loss ($I = 500$ A, $D = 8 \cdot 10^{-3}$ m).

thermal conductivity of tungsten and the small diameter of the cathodes considered (up to $1.5 \cdot 10^{-2}$ m).

The heat-balance equation of the cathode element takes the following form:

$$dQ_r = dQ_j + \frac{dQ_\lambda}{dx}\, dx. \tag{1}$$

The amount of heat lost by radiation can be written in the form

$$dQ_r = \varepsilon\sigma\pi D dx\,(T^4 - T_0^4). \tag{2}$$

For the plasmotron construction considered we can assume with reasonable accuracy that ε is equal to the degree of blackness of the cathode material.

The heat dissipated in the cathode element due to the passage of a current through it can be written as

$$dQ_j = I^2 R\,(T) = I^2 \rho_0 \frac{dx}{f}\,[1 + \beta\,(T - T_0)]. \tag{3}$$

The amount of heat transmitted by conduction and lost by radiation can be expressed in the form

$$\frac{dQ_\lambda}{dx}\, dx = \frac{d}{dx}\left(\lambda f \frac{dT}{dx}\, dx\right) = \lambda f \frac{d^2T}{dx^2}. \tag{4}$$

Then, taking Eqs. (2)-(4) into account, Eq. (1) takes the form

$$\frac{d^2T}{dx^2} = \frac{\varepsilon\sigma\pi D}{\lambda f}\,(T^4 - T_0^4) - \frac{I^2\rho_0\beta}{\lambda f^2}\,(T - T_0) - \frac{I^2\rho_0}{\lambda f^2} \tag{5}$$

or

$$\frac{d^2T}{dx^2} = aT^4 - bT + A. \tag{5a}$$

The differential equation of the form (5a) reduces to the first-order equation [5]

$$\frac{dT}{dx} = \sqrt{2\int (aT^4 - bT + A)\,dT + C_1}. \tag{5b}$$

The solution of this equation can be written in general form as

$$x = \int \frac{dT}{\sqrt{\frac{2aT^5}{5} - bT^2 + 2AT + C_1}} + C_2. \tag{6}$$

However, it is not possible to integrate this expression. It follows from Eqs. (5b) and (6) that

$$x = \int dT/(dT/dx) + C_2. \tag{7}$$

Putting $1/(dT/dx) = F(T)$, Eq. (7) can be rewritten in the form

$$x = \int F(T)\,dT + C_2. \tag{8}$$

Substituting arbitrary values of C_1 into Eq. (5b), we can obtain graphs of F(T) as a function of T, and it is then possible to approximate these curves by power functions of the form

$$F(T) = BT^m. \tag{8a}$$

Equation (8) then takes the form

$$x = \frac{B}{m+1} T^{m+1} + C_2', \tag{8b}$$

and for m = −1 we have

$$x = B \ln T + C_2', \tag{8c}$$

where C_2' is determined by the corresponding part of the approximation.

When finding F(T) as a function T and the solution of Eq. (8b), the temperature of the top of the cathode was taken to be the melting point of tungsten, while the temperature at the point of attachment of the cathode was taken to be 300°K, which was confirmed by experimental data [1].

The solution of Eqs. (5b)-(8b) enabled us to obtain the temperature gradients at the point of attachment of the cathode and in the arc "attachment" zone as a function of its length, and also the axial temperature distribution along the cathode.

As an example, we calculated $(dT/dx)_0$ as a function of l. In this case we took D = 0.012 m, λ = const = 100 W/m·deg, $\varepsilon = 0.30$, $\sigma = 5.7 \cdot 10^{-8}$ W/m^2·deg^4, $\rho_0 = 0.053 \cdot 10^{-6}\ \Omega \cdot$m, $\beta = 4.8 \cdot 10^{-3}$ deg^{-1}, and I = 600 A.

To check the correctness of this method we carried out experimental measurements of $(dT/dx)_0$ as the length of a cathode was changed. The temperature gradient was found from the equation $(dT/dx)_0 = Q_W/\lambda_0 f$.* The theoretical and experimental data are compared in Fig. 2, from which it is seen that for a ratio $l/D > 6$ good agreement is observed between the theoretical and experimental data.

The axial temperature distribution was calculated for a cathode of diameter 0.008 m and a current of 500 A. Figure 3 shows the results obtained (curve 2), compared with data given in [1] (curve 1), which is extrapolated to the calculated cathode length. As is seen from a comparison of these curves, neglect of the convective component of the heat loss for argon obviously has only a small effect on the axial temperature distribution along the cathode. The disagreement between the curves in the 1600-3000°K range is possibly due to the effect of hydrogen. But as is seen from Fig. 3, this disagreement is small for pressures below 20 torr.

The above method of calculation enables one to determine the temperature gradients along the cathode as a function of its length and to obtain the axial temperature distribution with an accuracy sufficient for practical purposes.

NOTATION

x	is the longitudinal coordinate of the rod, m;
Q_λ	is the heat quantity per unit time transferred by conduction, W;

*Q_W was found experimentally from calorimeter measurements of the water used to cool the cathode casing.

Q_r	is the radiation energy per unit time, W;
Q_j	is the Joule heat generation per unit time, W;
Q_p	is the heat quantity per unit time transferred from the arc to the cathode, W;
Q_w	is the heat quantity per unit time transferred from the cathode to casing, W;
T	is the temperature, °K;
D	is the cathode diameter, m;
l	is the cathode length, m;
I	is the plasmotron current, A;
0	is the initial section;
ε	is the emissivity of the material;
σ	is the Stefan—Boltzmann constant, $W/m^2 \cdot (deg)^4$;
ρ	is the specific electrical resistance, $\Omega \cdot m$;
f	is the cross-sectional area, m^2;
β	is the temperature coefficient of resistance, deg^{-1};
R	is the electrical resistance, Ω;
λ	is the thermal conductivity, $W/m \cdot deg$.

LITERATURE CITED

1. H. Huegel and G. Kruelle, "Phaemenologie und energiebilanz von lichtlogenkatoden bei niedrigen druecken und hohen stroemstaerken," Beit aus der Plasma Phys., 9, 2, 87 (1969).
2. G. P. Stel'makh, A. S. Sakhiev, and N. A. Chesnokov, "Application of plasma in chemical technology," Khim. Prom-st', No. 7 (1967).
3. V. I. Krylovich, "Temperature field in the cooled electrode of an electric-arc heater with a fixed arc," Inzh.-Fiz. Zh., No. 3 (1963).
4. A. L. Suris and S. Shorin, "Temperature field and heat flow in the hot cathode of an electric-arc heater," Inzh.-Fiz. Zh., 23, No. 6 (1972).
5. G. A. Korn and T. M. Korn, Manual of Mathematics, McGraw-Hill (1967).